# Relativistic and Neutrino Mass Effects in Partial Muon Capture [1]

by

Stanisław Ciechanowicz [2]

*Institute of Theoretical Physics, Wrocław University*
*PL 50-204 Wrocław, pl. Maksa Borna 9, Poland*

Zbigniew Oziewicz [3]

*Facultad de Estudios Superiores, Cuautitlán, UNAM*
*Apartado Postal # 25, C.P. 54700, Estado de México*

Nicolai Popov [4]

*Sektion Physik der Universität München*
*Schellingstr. 4, 80799 München, Germany*



**Abstract**

**The characteristics of the partial nuclear muon capture with massive left-handed Dirac neutrino and relativistic component of the muon wave function have been derived. The multipole amplitudes are given as a function of neutrino mass parameter and reduced nuclear matrix elements which are modified by the small component of the muon wave function. As an example, the capture rate, asymmetry and polarization of recoil nuclei are investigated in terms of these multipole amplitudes.**

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[1] Submitted for publication in Journal Phys. G

[2] e-mail: ciechano@proton.ift.uni.wroc.pl

[3] A member of Mexican Sistema Nacional de Investigadores
e-mail: oziewicz@servidor.unam.mx

[4] e-mail: popov@cip.physik.uni- muenchen.de

# 1 Introduction

Neutrino mass problem is a topic of many theoretical and experimental works. In recent years a considerable effort has been devoted to find massive neutrino in the different processes. It has been extensively looked for in neutrino oscillations, beta decay, double beta decay $\beta\beta$ (especially neutrinoless beta decay $\beta\beta(0\nu)$), charged lepton conversion as well as in cosmology and astrophysics (see e.g. [1- 4]).
Massive neutrinos are naturally predicted beyond the standard model of electroweak interaction, e.g. by "*see-saw*" mechanism. One can imagine also the gravitationally induced neutrino masses [5].
Experimental data of $\beta\beta$ and $\beta\beta(0\nu)$ decay give limits on the mass of Majorana neutrino and right-handed currents [6]. The solar neutrino puzzle can be explained in frame of *MSW* mechanism [7,8], or by long-wavelength vacuum oscillations [9].
As for muon capture process, the possibility to verify the existence of massive muon neutrino was proposed in ref. [10]. The calculations of nuclear muon capture observables with massive neutrino were undertaken in refs. [11,12]. So far, however, the calculations were performed without the small relativistic corrections which *a priori* could be competed with neutrino mass effect. We present here the results of our calculations of the observables of nuclear muon capture, taking into account both the neutrino mass effect and relativistic component of the muon wave function.

# 2 Effective Hamiltonian and multipole expansion

As earlier, we use the method of multipole expansion [13, 14] for the effective Hamiltonian in the nuclear muon capture on the lepton-hadron level, i.e. Fermi theory of four-fermion point interaction. The calculation is performed with relativistic muon and massive left-handed Dirac neutrino.

We take the effective $\mu$-capture Hamiltonian in the standard current $\times$ current form

$$< \mathbf{q}, \frac{1}{2}\nu, \hat{\mathbf{z}} \mid H_\mu \mid \frac{1}{2}m, \hat{\mathbf{z}} >= G J_\lambda l_\lambda, \tag{1}$$

where $G = \frac{G_\mu \cos\theta_c}{\sqrt{2}}$ is the coupling constant, $J_\lambda$ is the hadron current operator, $\mathbf{q}$ is vector of neutrino momentum, and

$$l_\lambda \;=\; \overline{\Psi}_{\nu_\mu}\gamma_\lambda(1 \;+\; \gamma_5)\Psi_\mu \tag{2}$$

is matrix element of the modified lepton current with the *massive left-handed Dirac neutrino*. With $\hat{\mathbf{z}}$, unit vector along $z$ axis, we denote the direction for lepton spin quantization.

The Dirac bispinor is commonly used for the neutrino wave function,

$$\Psi_{\nu_\mu} \equiv < \mathbf{r} \mid \mathbf{q}, \frac{1}{2}\nu, \hat{\mathbf{z}} >= \frac{1}{\sqrt{2}} \exp\{+i\mathbf{q}\cdot\mathbf{r}\} \begin{pmatrix} -c_1(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}) \\ c_0 \end{pmatrix} \otimes \chi_\nu, \tag{3}$$

where $c_0$ and $c_1$ are energy-mass coefficients,

$$c_o = \sqrt{1 + \frac{m_\nu}{E_\nu}}, \qquad c_1 = \sqrt{1 - \frac{m_\nu}{E_\nu}}. \tag{4}$$

We consider the nuclear partial transition in the muon capture process;

$$\mu^- + (A,Z)_{J_i} \rightarrow (A,Z-1)_{J_f} + \nu_\mu. \tag{5}$$

With the method of [13], we get the multipole expansion;

$$< -\mathbf{q}, J_f M_f; \mathbf{q}\frac{1}{2}h, \hat{\mathbf{q}} \mid H_\mu \mid J_i M_i; \frac{1}{2}\mu, \hat{\mathbf{q}} > \equiv \sum_{LM} i^{-L}\hat{L}\mathcal{T}_L^{h\mu} \left( \begin{array}{cc|c} J_i & L & J_f \\ M_i & M & M_f \end{array} \right) D^L_{M\mu-h}(\Omega_{\hat{\mathbf{q}}}). \tag{6}$$

The multipole amplitudes $\mathcal{T}_L^{h\mu}$ ($h, \mu = \pm\frac{1}{2}$, $\mid J_i - J_f \mid \leq L \leq J_i + J_f$ ) are given by linear combinations of the coupling constants and the reduced matrix elements in the nuclear, or in general, hadron space. With $\hat{\mathbf{q}}$, we denote that lepton spin is quantized now along the direction of neutrino momentum.

We find, that neutrino energy-mass coefficients factorize from the multipole amplitudes $\mathcal{T}_L^{h\mu}$ as follows;

$$\mathcal{T}_L^{h\mu} = \left(\frac{c_0 - 2hc_1}{2}\right) T_L^{h\mu}. \tag{7}$$

It should be noted that still $\mathcal{T}_L^{h\mu}$ are matrix elements of lepton current, where the diagonal terms $\mathcal{T}_L^{hh}$ are amplitudes with no spin-flip of the lepton in the weak current, while $\mathcal{T}_L^{h-h}$ are the amplitudes where the lepton spin is flipped in course of interaction. These amplitudes essentially differ for the neutrino helicities $h = +\frac{1}{2}$ and $h = -\frac{1}{2}$. We have, like in [13], the following expressions for the multipole amplitudes $T_L^{h\mu}$ in terms of the reduced matrix elements in hadron space;

$$T_L^{hh} = \tag{8}$$
$$[0LL, J_4]_{hh} + i[1LL, \mathbf{J}]_{hh} + \sqrt{\frac{L}{2L+1}}[1L-1L, \mathbf{J}]_{hh} + \sqrt{\frac{L+1}{2L+1}}[1L+1L, \mathbf{J}]_{hh},$$

$$T_L^{h-h} = \tag{9}$$
$$[0LL, J_4]_{h-h} - i[1LL, \mathbf{J}]_{h-h} + \sqrt{\frac{L+1}{2L+1}}[1L-1L, \mathbf{J}]_{h-h} + \sqrt{\frac{L}{2L+1}}[1L+1L, \mathbf{J}]_{h-h}.$$

Here, $[0LL, J_4]_{h\pm h}$ and $[1L'L, \mathbf{J}]_{h\pm h}$ are notations for the reduced nuclear matrix elements, in the sense of the Wigner-Eckart theorem.

For the relativistic muon in the $1s$ atomic state $(K - shell)$, we take the Dirac bispinor for the particle bound in the radial electrostatic potential;

$$\Psi_\mu = \begin{pmatrix} -iF(r)\chi_{1m}(\hat{\mathbf{r}}) \\ G(r)\chi_{-1m}(\hat{\mathbf{r}}) \end{pmatrix}, \tag{10}$$

with the customary definition of spherical spinors [15],

$$\chi_{km}(\hat{\mathbf{r}}) = \left[Y^l(\hat{\mathbf{r}}) \otimes \chi_{1/2}\right]_{Jm}, \qquad (J = l - \frac{1}{2}\mathrm{sign}k). \tag{11}$$

$G(r)$ and $F(r)$ are named as large and small radial components, respectively.

Next, these matrix elements are given explicitly in the product form of two terms, where the former is bilinear combination of the radial functions which emerge from the matrix elements of the leptonic current due to the neutrino plane wave and $1s$ muon bound state, and the latter is tensor operator that in the standard way couples the neutrino spherical harmonics and hadron current operators:

$$[0LL, J_4]_{hh} = \frac{\sqrt{2}}{\hat{J}_f} < f \parallel G\{G(r)j_L(qr) - 2hF(r)[\frac{L}{2L+1}j_{L-1}(qr) - \frac{L+1}{2L+1}j_{L+1}(qr)]\}J_4 Y^L(\hat{\mathbf{r}}) \parallel i >, \tag{12}$$

$$[0LL, J_4]_{h-h} = \frac{\sqrt{2}}{\hat{J}_f} < f \parallel G(-2h)F(r)\frac{\sqrt{L(L+1)}}{2L+1}[j_{L-1}(qr) + j_{L+1}(qr)]J_4 Y^L(\hat{\mathbf{r}}) \parallel i >, \tag{13}$$

$$[1LL, \mathbf{J}]_{hh} = \frac{\sqrt{2}}{\hat{J}_f} < f \parallel GF(r)\frac{\sqrt{L(L+1)}}{2L+1}[j_{L-1}(qr) + j_{L+1}(qr)][\mathbf{J} \otimes Y^L(\hat{\mathbf{r}})]^L \parallel i >, \tag{14}$$

$$[1LL, \mathbf{J}]_{h-h} = \frac{\sqrt{2}}{\hat{J}_f} < f \parallel G\{G(r)j_L(qr) - F(r)[\frac{L+1}{2L+1}j_{L-1}(qr) - \frac{L}{2L+1}j_{L+1}(qr)]\}[\mathbf{J} \otimes Y^L(\hat{\mathbf{r}})]^L \parallel i >, \tag{15}$$

$$[1L-1L, \mathbf{J}]_{hh} = \frac{\sqrt{2}}{\hat{J}_f} < f \parallel G(-2h)[G(r)j_{L-1}(qr) + 2hF(r)j_L(qr)][\mathbf{J} \otimes Y^{L-1}(\hat{\mathbf{r}})]^L \parallel i >, \tag{16}$$

$$[1L-1L, \mathbf{J}]_{h-h} = \frac{\sqrt{2}}{\hat{J}_f} < f \parallel G(-2h)[G(r)j_{L-1}(qr) + F(r)j_L(qr)][\mathbf{J} \otimes Y^{L-1}(\hat{\mathbf{r}})]^L \parallel i >, \tag{17}$$

$$[1L+1L, \mathbf{J}]_{hh} = \frac{\sqrt{2}}{\hat{J}_f} < f \parallel G(-2h)[G(r)j_{L+1}(qr) - 2hF(r)j_L(qr)][\mathbf{J} \otimes Y^{L+1}(\hat{\mathbf{r}})]^L \parallel i >, \tag{18}$$

$$[1L+1L, \mathbf{J}]_{h-h} = \frac{\sqrt{2}}{\hat{J}_f} < f \parallel G(-2h)[-G(r)j_{L+1}(qr) + F(r)j_L(qr)][\mathbf{J} \otimes Y^{L+1}(\hat{\mathbf{r}})]^L \parallel i >. \tag{19}$$

$j_v(qr)$ are the neutrino spherical Bessel functions. Tensor couplings of the hadron current spatial components $\mathbf{J}$ and spherical harmonics from the neutrino plane wave are as follows,

$$[\mathbf{J} \otimes Y^{L'}(\hat{\mathbf{r}})]^L_M = \sum_{\mu m} \left( \begin{array}{cc|c} 1 & L' & L \\ \mu & m & M \end{array} \right) (\mathbf{J})^1_\mu Y^{L'}_m(\hat{\mathbf{r}}). \tag{20}$$

# 3 Relativistic muon and massive neutrino

Question now is for the multipole amplitudes to compare the leading relativistic muon (RLMU) effects with bare massive neutrino (NEMA) contribution. To answer this, at least qualitatively in $0^{\pi_i} \xrightarrow{\mu^-} 1^{\pi_f}$ transitions, we have included RLMU effects into the formulae for multipole amplitudes calculated within the framework of the impulse approximation for the hadron current. Then, we generalize Fujii-Primakoff form factors to;

$$
\begin{aligned}
G_V^h &= g_V \left( 1 - 2h\tfrac{q}{2M} \right), \\
G_A^h &= g_A + 2hg_V(1+\mu_p-\mu_n)\frac{q}{2M}, \\
G_A^h - G_P^h &= -2hg_A + \left( g_A - g_P + 2\delta_{\frac{1}{2}h} g_P \tfrac{q}{m_\mu} \right) \frac{q}{2M}.
\end{aligned}
\tag{21}
$$

It is convenient for our present purposes to use point nucleus solution (Coulomb potential). Then the well known relation holds;

$$
F(r) = -\sqrt{\frac{1-\gamma}{1+\gamma}} G(r) \simeq -\frac{\alpha Z}{2} G(r), \tag{22}
$$

where $\gamma = \sqrt{1-(\alpha Z)^2}$. $\alpha$ is a fine structure constant and $Z$ - atomic number. We have considered contributions due to $F(r)$ together with derivative $G'(r)$.

Reduced matrix elements are written within the short notation;

$$
\{1L'L, j_l \boldsymbol{\sigma}\} \equiv \frac{\sqrt{2}}{\hat{J}_f} < f \| \sum_{i=1}^{A} G(r_i) j_l(qr_i) [\boldsymbol{\sigma}_i \otimes Y^{L'}(\hat{\mathbf{r}}_i)]^L \tau_i^- \| i > \tag{23}
$$

and additionally $\{1L'L, j_{L'}\boldsymbol{\sigma}\} \equiv [1L'L, \boldsymbol{\sigma}]$, if the labels in Bessel function and spherical harmonics are the same.
Below, the multipole amplitudes in $0^{\pi_i} \xrightarrow{\mu^-} 1^{\pi_f}$ transitions with the leading order RLMU contributions, $\sim \frac{\alpha Z}{2}$, are given;

$$
\begin{aligned}
iT_1^{hh}(\pi_i=\pi_f) &= (G_A^h - G_P^h)\sqrt{\frac{1}{3}} \left( [101, \boldsymbol{\sigma}] + \sqrt{2}[121, \boldsymbol{\sigma}] \right) + g_A[011, \boldsymbol{\sigma} \cdot i\frac{\mathbf{p}}{M}] \\
&\quad + \frac{\alpha Z}{2} g_A \sqrt{\frac{1}{3}} \left( \{101, j_1\boldsymbol{\sigma}\} - \sqrt{2}\{121, j_1\boldsymbol{\sigma}\} \right),
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
iT_1^{h-h}(\pi_i=\pi_f) &= -2hG_A^h \left( [101, \boldsymbol{\sigma}] - \sqrt{2}[121, \boldsymbol{\sigma}] \right) - g_V[111, i\frac{\mathbf{p}}{M}] \\
&\quad - 2h\frac{\alpha Z}{2} g_A \left( \{101, j_1\boldsymbol{\sigma}\} - \sqrt{2}\{121, j_1\boldsymbol{\sigma}\} \right),
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
T_1^{hh}(\pi_i=-\pi_f) &= G_V^h[011, \mathbf{1}] + 2hg_V\sqrt{\frac{1}{3}} \left( [101, i\frac{\mathbf{p}}{M}] + \sqrt{2}[121, i\frac{\mathbf{p}}{M}] \right) \\
&\quad + \frac{\alpha Z}{2} \cdot \frac{1}{3} \left( 2hg_V\{011, j_0\mathbf{1}\} - g_A\sqrt{2}\{111, j_0\boldsymbol{\sigma}\} \right),
\end{aligned}
\tag{26}
$$

$$T_1^{h-h}(\pi_i=-\pi_f) = -G_A^h[111,\boldsymbol{\sigma}] + 2hg_V\left([101, i\frac{\mathbf{p}}{M}] - \sqrt{2}[121, i\frac{\mathbf{p}}{M}]\right) + \frac{\alpha Z}{2}\left(2hg_V\frac{\sqrt{2}}{3}\{011, j_0\mathbf{1}\} + g_A 2\{111, j_0\boldsymbol{\sigma}\}\right). \quad (27)$$

The largest RLMU contributions are given here. These are only from the small radial component of the muon function $F(r)$. All reduced matrix elements with Bessel functions of rank $L-2$ and $L-3$ have been omitted as they can not contribute for multipolarity $L=1$.

Estimating NEMA contributions we perform with Lommel integral for spherical Bessel functions for $(q<E)$ in case of *massive* , and $(q=E)$ *massless* neutrinos;

$$I_v(q) = \int_0^\infty j_v(qr)dr = \frac{\sqrt{\pi}}{2}\cdot\frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v+2}{2})}\cdot\frac{1}{q} \quad (28)$$

With this, the difference between $j_v(qr)$ and $j_v(Er)$ in radial integral with nuclear wave functions can be evaluated;

$$I_v(q) = \frac{1}{c_0c_1}I_v(E) \simeq \left(1+\frac{1}{2}\frac{m_\nu^2}{q^2}\right)I_v(E) \quad (29)$$

Multipole amplitudes $T_L^{h\mu}$ change approximately by a global factor $(1+\frac{1}{2}\frac{m_\nu^2}{q^2})$. We take the upper limit $m_\nu c^2 \le 160\ keV$ from [16], and $q \simeq 100MeV$. The magnitude of $\frac{1}{2}\frac{m_\nu^2}{q^2} = 1.28\times10^{-6}$ measures the change of the amplitudes. Coulomb coupling parameter $\alpha Z$ for e.g. $^{12}C$ gives $\frac{1}{2}\alpha Z \simeq 2.16\times10^{-2}$. This suggests RLMU effect to the multipole amplitudes by four orders of magnitude stronger than NEMA contribution.

# 4 Observables in muon capture

In order to obtain the capture rate and correlation characteristics in muon capture, we use the polarization density matrix for the final state of the system,

$$\begin{aligned} &< -\mathbf{q}, J_fM_f; \mathbf{q}, \frac{1}{2}\nu \mid \rho_f \mid -\mathbf{q}, J_fM_f^{'}; \mathbf{q}, \frac{1}{2}\nu^{'} >= \quad (30)\\ &= \frac{1}{2(2J_i+1)}\sum_{M_i,m,m^{'}} < -\mathbf{q}, J_fM_f; \mathbf{q}, \frac{1}{2}\nu \mid H_\mu \mid J_iM_i; \frac{1}{2}m >< \frac{1}{2}m \mid 1+2\mathbf{P}_\mu\cdot\mathbf{s}_\mu \mid \frac{1}{2}m^{'} > \\ &\quad < -\mathbf{q}, J_fM_f^{'}; \mathbf{q}, \frac{1}{2}\nu^{'} \mid H_\mu \mid J_iM_i; \frac{1}{2}m^{'} >^*, \end{aligned}$$

where direction of $\hat{\mathbf{z}}$ is the spin quantization axis for both nucleus and leptons. $\mathbf{P}_\mu$ is the residual muon polarization on the K-orbit and $\mathbf{s}_\mu = \frac{1}{2}\boldsymbol{\sigma}$ denotes the muon spin operator. Averaging over the initial states and summing over the final ones, which are not measured in the experiment, we obtain the formulae for nuclear muon capture rate, $\Lambda_c \sim Tr\overline{\rho}_f \equiv \frac{1}{4\pi}\int d\hat{\mathbf{P}}_\mu \frac{1}{4\pi}\int d\hat{\mathbf{q}}\, Tr\rho_f$, recoil polarization vector $\mathbf{P}_f \equiv Tr\left(\mathbf{J}_f\rho_f\right)/J_f Tr\overline{\rho}_f$, and the recoil asymmetry $\mathcal{W}(\theta) \equiv Tr\rho_f/Tr\overline{\rho}_f$.
The formula for muon capture rate is;

$$\Lambda_c = c_0c_1N_{fi}\frac{2J_f+1}{2(2J_i+1)}\sum_h\left(\frac{1-2hc_0c_1}{2}\right)\lambda_h, \quad (31)$$

here $c_0c_1N_{fi}$ is a phase space volume, with conventional factor for $m_\nu = 0$;

$$N_{fi} = \frac{1}{2\pi}q^2\left[1 + \frac{m_\mu}{\sqrt{m_\mu^2 + (Am_p)^2}}\right]^{-1}. \tag{32}$$

$q$ is absolute value of the neutrino momentum, and $c_0c_1 = [1 + (m_\mu/q)^2]^{-1/2}$. $A$ is the nucleus mass number, while $m_\mu$ and $m_p$ are muon and proton masses, respectively. All the multipole amplitudes come into the coefficient,

$$\lambda_h = \sum_L \left(\mid T_L^{hh}\mid^2 + \mid T_L^{h-h}\mid^2\right). \tag{33}$$

Then, we define the weights $\omega_h$;

$$\omega_h = \left(\frac{1 - 2hc_0c_1}{2}\right)\lambda_h\left[\left(\frac{1 + c_0c_1}{2}\right)\lambda_- + \left(\frac{1 - c_0c_1}{2}\right)\lambda_+\right]^{-1}. \tag{34}$$

These weights correspond respectively to the contributions of the neutrinos with $h = +\frac{1}{2}$ and $h = -\frac{1}{2}$ to the observables. In the lowest order approximation for the energy-mass coefficients, eq. (4), (with $m_\nu/q \ll 1$) we get $\omega_- \simeq 1$, and $\omega_+ \simeq 0$.
Here we give the recoil polarization formula in the $J_i = 0 \xrightarrow{\mu^-} J_f = 1$ transitions;

$$\mathbf{P}_f = \sum_h \omega_h \mathbf{P}_f(h), \tag{35}$$

where

$$\mathbf{P}_f(h) = [-2hA_h + (A_h - ReB_h)(\mathbf{P}_\mu \cdot \hat{\mathbf{q}})]\,\hat{\mathbf{q}} + ReB_h\mathbf{P}_\mu + 2hImB_h(\mathbf{P}_\mu \times \hat{\mathbf{q}}). \tag{36}$$

The structure functions $A_h$ and $B_h$ are expressed in terms of the multipole amplitudes ratio;

$$\frac{T_1^{hh}}{T_1^{h-h}} = \sqrt{\frac{1}{2}}x_h \exp[i\phi_h], \tag{37}$$

$$A_h = \frac{2}{2 + x_h^2}, \qquad B_h = \frac{2x_h\exp[i\phi_h]}{2 + x_h^2}. \tag{38}$$

For the longitudinal polarization, $P_L = \frac{1}{4\pi}\int d\hat{\mathbf{q}} < \mathbf{P}_f \cdot \hat{\mathbf{q}} >$, we get

$$P_L = \sum_h \omega_h 2hA_h. \tag{39}$$

Next we consider angular asymmetry of recoil with respect to the direction of the muon polarization;

$$\mathcal{W}(\theta) = 1 + \sum_h \omega_h 2h\alpha_h \hat{\mathbf{q}} \cdot \mathbf{P}_\mu, \tag{40}$$

where $P_\mu \cos\theta = -\hat{\mathbf{q}} \cdot \mathbf{P}_\mu$. We express the asymmetry coefficients $\alpha_h$ as the functions of $x_h$ and find their relation to recoil polarization coefficient $A_h$;

$$\alpha_h = \frac{2 - x_h^2}{2 + x_h^2} = 2A_h - 1. \tag{41}$$

With the results of [12] for the neutrino polarization, we present simple example of the link between $CP$-violation and massiveness of the Dirac neutrino. We have calculated in accordance with the definition $< \mathbf{s}_\nu >\equiv Tr\left(\mathbf{s}_\nu \rho_f\right)/Tr\overline{\rho}_f$;

$$< \mathbf{s}_\nu >= \left[a + (c - Reb)(\mathbf{P}_\mu \cdot \hat{\mathbf{q}})\right]\hat{\mathbf{q}} + Reb\mathbf{P}_\mu + Imb\hat{\mathbf{q}} \times \mathbf{P}_\mu \ , \tag{42}$$

where structure functions are given by,

$$a = \sum_h h\omega_h, \qquad c = -\frac{1}{2}\sum_h \omega_h \alpha_h, \tag{43}$$

$$b \ = \ c_0 c_1 \frac{m_\nu}{q}\frac{1}{2}T_1^{--}T_1^{++*}\left[\left(\frac{1 + c_0 c_1}{2}\right)\lambda_- + \left(\frac{1 - c_0 c_1}{2}\right)\lambda_+\right]^{-1}. \tag{44}$$

(Since then, we denote $h \equiv sign\, h$ and $\mu \equiv sign\, \mu$).
So that, observing in muon capture $< \mathbf{s}_\nu \cdot \hat{\mathbf{q}} \times \mathbf{P}_\mu >\sim Imb \neq 0$ would indicate both $CP$-violation and massive neutrino. For the longitudinal neutrino polarization we have;

$$< \mathbf{s}_\nu \cdot \hat{\mathbf{q}} >= a - cP_\mu \cos\theta, \tag{45}$$

which gives for $m_\nu = 0$ the recoil asymmetry; $< \mathbf{s}_\nu \cdot \hat{\mathbf{q}} >= -\frac{1}{2}\mathcal{W}_0(\theta)$. Averaging this over $\mathbf{P}_\mu$, we get the helicity for the left-handed massless neutrino; $< \mathbf{s}_\nu \cdot \hat{\mathbf{q}} >_{av}= -\frac{1}{2}$.

# 5 Discussion and conclusions

In the model with massive left-handed neutrino, [12], the existence of the four independent multipole amplitudes was obtained (doubled in comparison to the massless case) in the nonrelativistic approximation of the muon wave function. As follows from our consideration the calculation with accurate muon wave function does not change the number of multipole amplitudes, which is still four. So the form of the kinematic characteristics of the muon capture expressed by multipole amplitudes remains the same. However, the numerical predictions will change, as the number of nuclear matrix elements in the multipole amplitudes is larger owing to relativistic muon contribution. There appear additional nuclear matrix elements $[1LL, J]_{hh}$ and $[0LL, J_4]_{h-h}$. Moreover, the structure of other nuclear matrix elements is also different due to additional terms proportional to $F(r)$ and $G'(r)$.

Considering nuclear observables, we obtain strict results in terms of the multipole amplitudes. In addition to standard term, there is another one with the amplitudes for positive helicity of neutrino. This term multiplies by the weight $\omega_+$, which is proportional to $\frac{m_\nu^2}{4q^2}$. In these observables the multipole amplitudes with opposite neutrino helicities do not interfere.

Additionally , NEMA effects contribute to neutrino Bessel functions in the multipole amplitudes and very weakly to the phase space volume. From these, the effect to Bessel

function shifts the multipole amplitudes by a term proportional to $\frac{1}{2}\frac{m_\nu^2}{q^2}$, and doubles its full value in the capture rate.

On the contrary, the ratio of the multipole amplitudes,

$$T_1^{hh}/T_1^{h-h}, \tag{46}$$

weakly depends on NEMA and RLMU effects, which practically disappear in allowed transitions, i.e. for $\pi_i = \pi_f$. In conclusion, such observables as recoil polarization, recoil asymmetry etc., to a high degree are independent of these effects.

Qualitative analysis suggests relativistic muon effects larger by the four orders of magnitude than massive neutrino effects in light nuclei. In progress, there is our analysis on specific nuclei for quantitative results. Situation is different in muon capture reactions with three-body final states, like e.g.;

$$(A, Z)_{J_i} + \mu^- \to (A-1, Z-1)_{J_f} + n + \nu_\mu, \tag{47}$$

There are experiments performed and planned,

$$\begin{aligned} \mu^- \ + \ ^2H \ &\to \ \nu_\mu + 2n, \\ \mu^- \ + \ ^3He \ &\to \ \nu_\mu \ + \ ^2H + n, \end{aligned}$$

to measure high energy-momentum transfer to hadrons, which automatically gives low energy neutrino, $m_\nu/E_\nu \to 1$. Eventual analysis of the data with the massive neutrino theory could be possible.

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